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ABSTRACT

This collection of lessons presents units designed to teach specific mathematical and genetic concepts in the context of biomedical problems. Twenty-five lessons are presented with a review unit concluding the collection. Each lesson presents: (1) objectives; (2) periods recommended; (3) overview and remarks; and (4) a key to the problem set presented in the student text. (RE)

BIOMEDICAL MATHEMATICS

UNIT V

INTRODUCTION TO LOGARITHMS, THE BINOMIAL THEOREM AND GENETICS

INSTRUCTOR'S MANUAL
REVISED VERSION, 1976

THE BIOMEDICAL INTERDISCIPLINARY CURRICULUM PROJECT

SUPPORTED BY THE NATIONAL SCIENCE FOUNDATION

TABLE OF CONTENTS

LESSON 1: THE LAWS OF EXPONENTS

OBJECTIVES:

The student will:

- use the additive law of exponents to solve problems involving integral exponents.
- use the multiplicative law of exponents to solve problems involving integral exponents.
- use and interpret the zero exponent correctly.
- solve problems involving negative integral exponents.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

Mathematics Unit V covers the following two subject areas:

Lessons 1 - 9: Introduction to Logarithms
Lessons 10 - 26: Genetics.

There is one important interdisciplinary relationship with Biomedical Science which you should keep in mind. Mathematics Lesson 22 is predicated on a knowledge of the terminology of classical genetics. This terminology is introduced in or before Science Lesson 16. Therefore it would be ideal for Mathematics Lesson 22 to be presented after Science Lesson 16. If this is not feasible, you will need to introduce the terminology in Mathematics class (or the Biomedical Science Instructor might be willing to take a few minutes of class time to develop the needed concepts). Also it would be very useful for you to read Science Sections 16 - 18, 20, 21 and the "floating" sequence on population genetics. These readings will help you to present the Mathematics lessons using terminology similar to that used by the Science instructor.

The Science "floating" sequence X on population genetics is intended to be started immediately following the Mathematics Lessons on population genetics (Lessons 23 - 25). Science sequence X can be taught any time after Science Lesson 20. If Mathematics Lesson 22 is presented soon after Science Lesson 16, then the tie with the "floating" sequence will present no difficulties. If it is presented before Science Lesson 16, there will be a gap between the two treatments of population genetics. This, however, will present no problems in subsequent Mathematics lessons.

The interdisciplinary connections can be displayed as follows.

MATHEMATICS		Lesson 22	Lesson 25	
SCIENCE	Lesson 16			Floating Sequence X

There are no specific interdisciplinary ties with Biomedical Social Science in this unit. However, there are several problems in Mathematics Lessons 18 and 19 that explore patterns of mating with respect to race and religion. This material provides a natural jumping-off place for discussions in Social Science class, if desired. The concept of race is discussed in Science in the "floating" sequence on population.

Lessons 1 - 9 of Mathematics Unit V present an introduction to logarithms. This material bears no interdisciplinary relationship with the Science course. It is intended to provide a basic foundation such as is needed for college entrance examinations. Because hand calculators are replacing logarithms in the performance of calculations, the treatment emphasizes the logarithmic properties rather than calculations. The rich area of exponential and logarithmic <u>functions</u> will be explored in Mathematics Unit VII. The lessons on logarithms can also be used to sychronize the Mathematics and Science courses. Depending upon your particular pacing needs, the logarithm material can be presented anytime before Unit VII.

Lesson 1 of this sequence is intended to provide a grounding in the laws of exponents. These laws are necessary to the derivation of the basic logarithmic properties.

KEY--PROBLEM SET 1:

1. c	9. 0	17. 12	25. 1	33. $\frac{1}{9}$
2. 6	10. 5	18. 6	26. 3	34. $\frac{1}{16}$
3. 5	11. a	19. 3	27. 2	35. 10
4. 8	12. 12	20. 12	28. 4	36. 14
5. 8	13. 12	21. 5	29. 6	37. 12
6. 6	14. 20	22. 3	30. $\frac{1}{4}$	38. 22
7. 3	15. 10	23. 1	31. $\frac{1}{8}$	39. 24
8. 10	16. 6	24. 1	32. $\frac{1}{3}$	

LESSON 2: FRACTIONAL EXPONENTS

<u>OBJECTIVES</u>:

The student will:

- use the laws of exponents in solving problems involving fractional exponents.
- convert an expression of the form $A^{\frac{1}{n}}$ to the equivalent form $\sqrt[n]{A}$.

PERIODS RECOMMENDED:

One

KEY--PROBLEM SET 2:

1. 1	11. $\sqrt[6]{3}$	21. 4	31. $\frac{2}{3}$
2. 1	12. $\sqrt[30]{48}$	22. .1	32. $\frac{1}{3}$
3. 1	13. 10	23. .9	33. $\frac{1}{4}$
4. $\frac{1}{2}$	14. 6	24. $\approx (1.35)^2$	34. $-\frac{2}{3}$
5. $\frac{1}{4}$	15. 4, 11	25. $\approx (1.35)^5$	35. $-\frac{1}{3}$
6. $\frac{2}{3}$	16. 3	26. $\approx (1.03)^3$	36. $\approx \frac{1}{(1.97)^2}$
7. $\sqrt{8}$	17. 3	27. $\approx (1.03)^8$	37. $\approx \frac{1}{(1.97)^3}$
8. $\sqrt{10}$	18. 2	28. $\approx (1.35)^2(1.03)^8$ or $\approx (1.03)^{28}$	38. $\approx \frac{1}{(1.97)^5}$
9. $\sqrt[3]{4}$	19. $\frac{1}{4}$	29. $\approx (1.35)^5(1.03)^3$ or $\approx (1.03)^{53}$	39. $\approx \frac{1}{(1.97)^7}$
10. $\sqrt[4]{11}$	20. $\frac{4}{5}$	30. $\frac{1}{2}$	

LESSON 3: COMMON LOGARITHMS

OBJECTIVES:

The student will:

- estimate values of 10^x graphically.
- estimate values of log y graphically.
- determine log y when y is an integral power of ten.
- solve problems requiring the use of the identities $\log (10^x) = x$ and $10^{\log y} = y$.

PERIODS RECOMMENDED:

Two

OVERVIEW AND REMARKS:

A graph resembling that of $y = 10^x$ will be generated from counts of bacterial colonies in Biomedical Science Laboratory Activity 10. In Mathematics Unit VII such relationships will be explored in more detail.

KEY--PROBLEM SET 3:

1.	6.8	13.	.26	25.	a. −3 b. −3	37.	.83	49.	$x = b$
2.	1.6	14.	.43	26.	3	38.	$x = 2$	50.	$x = b$
3.	9.5	15.	.58	27.	5	39.	$x = 5$	51.	$y = \frac{5}{2}$
4.	4.8	16.	.12	28.	−5	40.	$x = 5$	52.	$y = 2$
5.	2.8	17.	.91	29.	−7	41.	$x = 7$	53.	$y = 27$
6.	1.4	18.	.62	30.	.7	42.	$x = 9$	54.	$y = \frac{5}{2}$
7.	7.6	19.	.36	31.	.35	43.	$x = .5$	55.	$y = \pm 5$
8.	2.4	20.	.87	32.	.89	44.	$x = a$	56.	$y = 16$
9.	5.6	21.	0	33.	243.2	45.	$x = m$	57.	$y = \frac{5}{6}$
10.	3.2	22.	1	34.	21	46.	$x = 4$	58.	$x = 21$
11.	.09	23.	a. 2 b. 2	35.	12	47.	$x = 3$	59.	$x = 2$
12.	.79	24.	a. −2 b. −2	36.	80	48.	$x = 3$	60.	$y = 8$

LESSON 4: THE LOGARITHMIC PROPERTIES

OBJECTIVE:

The student will solve problems involving the following identities:

$$\log (yz) = \log y + \log z$$

$$\log (y^{v}) = v \log y$$

$$\log \frac{1}{y} = -\log y$$

$$\log \frac{z}{y} = \log z - \log y$$

PERIODS RECOMMENDED:

One

KEY--PROBLEM SET 4:

1.	b	10.	2	19.	2	28.	4	37.	6
2.	3	11.	8	20.	2	29.	16	38.	.78
3.	.003	12.	9	21.	4	30.	2	39.	1.55
4.	13	13.	15	22.	d	31.	5	40.	.90
5.	14	14.	.2	23.	2	32.	4	41.	1.18
6.	10	15.	2	24.	8	33.	80	42.	1.20
7.	5	16.	3	25.	$\frac{2}{3}$	34.	2	43.	1.16
8.	4	17.	8	26.	17	35.	11	44.	.80
9.	d	18.	2	27.	97	36.	$\frac{10}{7}$	45.	.15

46. $\log 19 - \log 5 \simeq .58$

47. $\log 3 + 2 \log 2 \simeq 1.08$

48. $(10^x)^v = 10^{xv}$

Let $y = 10^x$ or $x = \log y$

Then

$$y^v = 10^{v \log y}$$

And

$$\log y^v = \log (10^{v \log y}) = v \log y$$

49. $\frac{1}{10^x} = 10^{-x}$

Let $y = 10^x$ or $x = \log y$

Then

$$\frac{1}{y} = 10^{-\log y}$$

And

$$\log \frac{1}{y} = \log (10^{-\log y}) = -\log y$$

LESSON 5: USING A TABLE OF LOGARITHMS

OBJECTIVE:

The student will use a logarithm table to find logarithms and antilogarithms.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

The logarithm table used in this lesson is on the inside back cover of the Student Text. A copy is included in the same location in this manual for your convenience.

KEY--PROBLEM SET 5:

1.	1	9.	5	17.	log 2.81 + 2	25.	3.973
2.	3	10.	-2	18.	log 2.02 + (-3)	26.	4.763
3.	-6	11.	2	19.	.851	27.	2.914
4.	4	12.	-1	20.	.663	28.	1.633
5.	-5	13.	log 8.6 + 3	21.	.934	29.	-3.854
6.	11	14.	log 2.4 + (-2)	22.	.380	30.	-.678
7.	-2	15.	log 7.3 + 1	23.	.982	31.	-1.409
8.	-2	16.	log 1.12 + (-4)	24.	.255	32.	-2.237
						33.	-4.114

34. characteristic = 2 mantissa = .886 y ≈ 770

35.	c = 3	m = .613	y ≈ 4100	43.	c = -3	m = .362	y ≈ .0023
36.	c = 1	m = .964	y ≈ 92	44.	c = 1	m = .883	y ≈ 76
37.	c = 4	m = .826	y ≈ 67000	45.	c = 2	m = .125	y ≈ 130
38.	c = 2	m = .041	y ≈ 110	46.	c = 5	m = .240	y ≈ 170,000
39.	c = -6	m = .778	y ≈ .000006	47.	c = -3	m = .969	y ≈ .0093
40.	c = -3	m = .531	y ≈ .0034	48.	c = -4	m = .311	y ≈ .0002
41.	c = -2	m = .431	y ≈ .027	49.	c = -2	m = .572	y ≈ .037
42.	c = -4	m = .756	y ≈ .00057				

50. a. .041
    b. .41
    c. y ≈ 2.6

51. .91

52. .40

53. .0001

LESSON 6: APPLICATIONS OF LOGARITHMS

OBJECTIVES:

The student will:

- use logarithms to solve pH problems.
- use logarithms to solve fractional exponent problems.

PERIODS RECOMMENDED:

One or two

KEY--PROBLEM SET 6:

1. c	6. 11.51	11. 6.6×10^{-4}M	16. 6.9×10^{-3}M
2. b	7. 1.33	12. 5.5×10^{-6}M	17. 3.1×10^{-7}M
3. 6	8. 0.16	13. 2.4×10^{-8}M	18. 5.4×10^{-5}M
4. 3.08	9. 2.70	14. 3.8×10^{-10}M	
5. 8.26	10. 4.16	15. 4.7×10^{-9}M	

19. The x column corresponds to the log y column in the log table and the 10^x column corresponds to the y column. All of the ordered pairs in the decimal power table appear in the log table except (.1, 1.25) and (1.0, 10.0), but the values of x have been rounded to the nearest tenth in the decimal power table.

20. 4	25. 2.16	30. 10.00	35. $x = \frac{\log 5}{3 \log 8}$
21. 8	26. 2.79	31. 38.19	36. $x = \frac{1}{\log 4}$
22. .29	27. 4.66	32. $x = \frac{\log 3}{\log 11}$	37. $x = \frac{\log 2}{\log 6 - 2 \log 3}$
23. 1.27	28. 6.03	33. $x = \frac{\log 31}{\log 5}$	38. $x = 10^{\frac{\log 21}{.55}}$
24. 1.67	29. 7.78	34. $x = \frac{2}{\log 2}$	39. $x = 10^{\frac{2}{1.31}}$

LESSON 7: MORE LOGARITHMIC APPLICATIONS

OBJECTIVES:

The student will:

- compute the basal metabolic rate for an average person of his or her height and mass.
- solve problems relating the power ratio of sounds to their intensity difference in decibels.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

Problem 7 in the problem set requires reference to the Absorbance-Transmittance Table, which is located inside the back cover of the Science Laboratory Manual.

KEY--PROBLEM SET 7:

1. Answer will depend on student's height and weight.
2. 10,000
3. 1,000,000
4. 100 decibels

5. 120 decibels

6. 7.1 x 10^{10}

7. Absorbance = 2 - log (Transmittance)

LESSON 8: CONSTRUCTING A SLIDE RULE

OBJECTIVE:

The student will use a graph of x = log y to construct a slide rule.

PERIODS RECOMMENDED:

One or more

SUPPLIES:

Straightedge (one per student)

Slide rules (a few for comparison with student-made slide rules)

OVERVIEW AND REMARKS:

The process of constructing slide rules is outlined in Section 8 of the Student Text. You will need to make a decision about the length of the vertical scale. If the scale has the same length as that on the slide rules you bring to class it will facilitate comparison. A length of 25 cm is common for commercial slide rules. However, keep in mind that the graphing will be simpler if the vertical scale has a length of 20 cm. The main factor to consider is class proficiency in graphing.

Once the student slide rules have been constructed, the following activities can be performed.

1. Simple multiplications, e.g., 2 × 4, 2 × 3, to illustrate multiplication.

2. Simple division, e.g., 4 ÷ 2, 8 ÷ 4, to illustrate division.

3. Calculations can be performed which will allow more locations to be labeled on the scale. For example, 7 ÷ 2 will locate 3.5 and 9 × 8 will locate 7.2. This is a mechanical equivalent of the process used in Problems 38-47 of Section 4.

4. Comparison of student slide rules with the ones you brought to class. These activities can be used as a jumping-off point for a more detailed treatment of the slide rule if you wish. You might explore this possibility if you are too far ahead of Science Class.

LESSON 9: REVIEW

OBJECTIVE:

The student will solve problems related to the objectives for Lessons 1 - 8.

PERIODS RECOMMENDED:

One or two.

OVERVIEW AND REMARKS:

Probelms 48 and 49 involve the relationship between heart rate and body weight for animals. This relationship will be explored in more detail in Mathematics Unit VII.

Some interesting remarks on heart rate and longevity for animals can be found in the last chapter of the following book: Isaac Asimov, The Human Body, Its Structure and Operation, Houghton Mifflin Co., Boston, 1963.

KEY--REVIEW PROBLEM SET 9:

1. 9	13. 2	25. 2	37. 9.7
2. 4	14. .1	26. 6	38. 620
3. 10	15. $\frac{3}{4}$	27. 2	39. .0018
4. 4	16. T	28. 8	40. 7.68
5. 5	17. F	29. 2	41. 5.14
6. 22	18. T	30. .6	42. 6.6×10^{-7} M
7. 1	19. T	31. .58	43. 2.4×10^{-10} M
8. 1	20. 2	32. 15	44. 2.1
9. $\frac{1}{25} = .04$	21. -4	33. 2	45. 6.3
10. $\frac{1}{8} = .125$	22. 5	34. .924	46. $x = \frac{\log 82}{\log 3}$
11. 1	23. .86	35. 3.580	47. $x = 10^{\frac{3}{3.2}}$
12. $\frac{1}{3}$	24. 329	36. -3.367	

48. 570 $\frac{\text{beats}}{\text{min}}$ if the technique of Section 7-1 is used.

49. 28 $\frac{\text{beats}}{\text{min}}$ again using the technique of Section 7-1.

A simple explanation for the relationship between heart rate and mass is the following: the ratio of surface area to mass is larger for small animals. Since body heat is lost through the surface, the heat loss per unit body mass is greater for small animals. This loss must be compensated by a higher metabolic rate, and a higher heart rate is one consequence.

50. 100,000

51. 25.8 dB

LESSON 10: INTRODUCTION TO GENETICS

OBJECTIVE:

The student will use path diagrams to count the number of possible ways in which several different objects or traits can occur together.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

This lesson begins the treatment of mathematical ideas in genetics. The first five lessons in the sequence develop the ideas of patterns, permutations and combinations.

If you wish, you might relate binary counting and the listing of all patterns of a certain length. For example, the list on page 35 of the Student Text can be obtained by writing the numbers 0-7 in binary notation and then identifying f with the digit 0 and m with the digit 1.

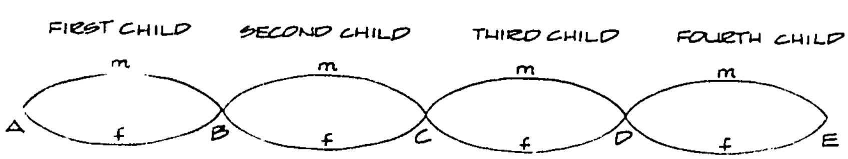
000 - fff	100 - mff
001 - ffm	101 - mfm
010 - fmf	110 - mmf
011 - fmm	111 - mmm

There are a number of problems in the next few problem sets which refer to a standard deck of cards. It would be a good idea to have a deck of cards on hand in case some explanation is needed.

Note that Problem 12b in the problem set calls for the use of logarithms.

KEY--PROBLEM SET 10:

1. 2
2. 2
3. 2 x 2
4. 2
5. ff fm mf mm
6. 3
7.

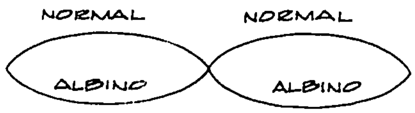


8. 16
9. 2^4
10.

ffff	fmff	mfff	mmff
fffm	fmfm	mffm	mmfm
ffmf	fmmf	mfmf	mmmf
ffmm	fmmm	mfmm	mmmm

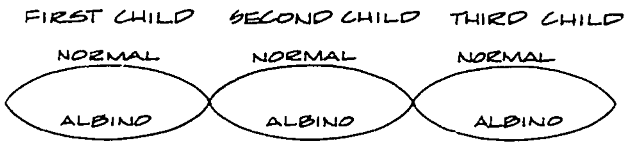
11. 8
12. a. 2^{69}
    b. 5.9×10^{20}

13. a.



b.

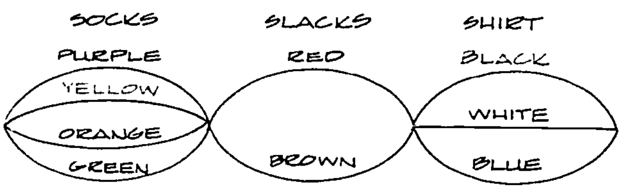
14. a.



b.

15. $2^5 = 32$ 16. $26^2 = 676$ 17. $26^3 = 17,576$ 18. $17 \times 8 = 136$

19. a.



b. $4 \times 2 \times 3 = 24$

20. $13 \times 13 \times 13 \times 13 = 28,561$

21. $2^4 = 16$

22. $9 \times 10^3 = 9000$

23. $2^7 = 128$

24. $9 \times 10^6 = 9,000,000$

25. $26^2 \times 10^3 = 676,000$

LESSON 11: PATTERNS OF TRAITS

OBJECTIVE:

The student will compute the total number of possibilities for a combination of traits, given the number of possibilities for each trait.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

The concept of a genetic wheel is introduced in this lesson. A more complex genetic wheel will be used in Science Lesson 15.

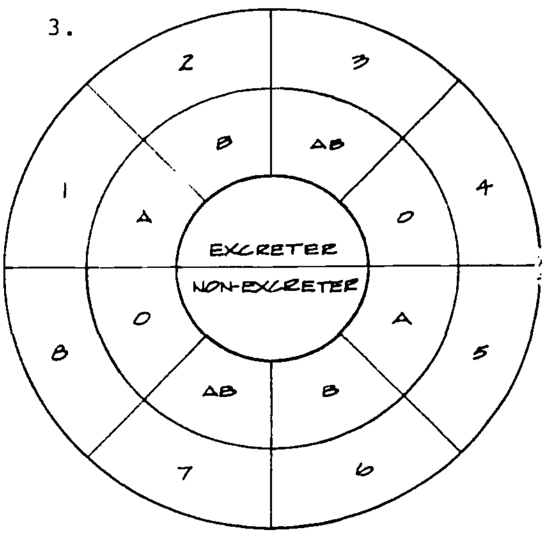
Problem 16 concerns chromosome sorting in meiosis. This subject is discussed in Science Lesson 3. Also, note that Problem 16b requires the use of logarithms.

KEY--PROBLEM SET 11:

1. $2 \times 4 = 8$

2. E,A E,B E,AB E,O
   N,A N,B N,AB N,O

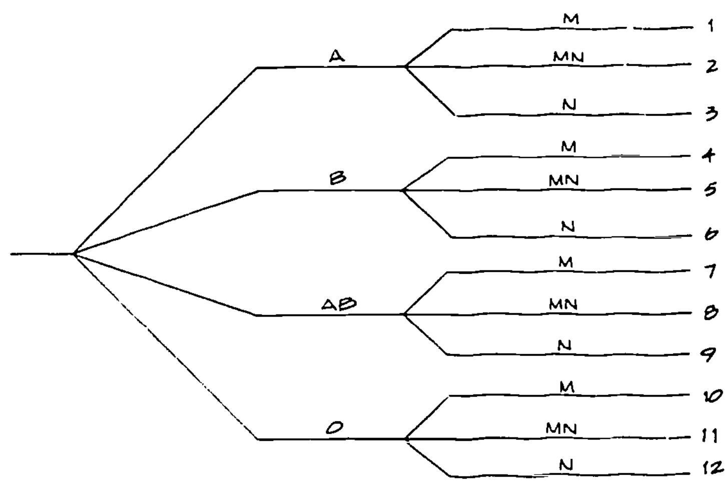
3.



4. are

5. $4 \times 3 = 12$

6.



7. $2 \times 4 \times 3 \times 2 = 48$

8. $2 \times 2 \times 3 \times 2 = 24$

9. $2 \times 4 \times 1 \times 2 = 16$

10. $2 \times 4 \times 3 \times 1 = 24$

11. a. 16
    b. 16
    c. $16 \times 16 = 256$

12. $5 \times 5 = 25$

13. $3 \times 3 = 9$

14. $2 \times 3 = 6$

15. There are 5, listed below. An "X" indicates presence of the ability. "O" indicates absence.

	Curling	Folding	Cloverleaf
1	O	O	O
2	X	O	O
3	X	X	O
4	X	O	X
5	X	X	X

16. a. 2^{23}
    b. 8.4×10^{6}

LESSON 12: PERMUTATIONS

OBJECTIVES:

The student will compute the number of permutations of n things taken n at a time, and will use factorial notation as an aid in such computations.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

Section 12-1 contains a list of all permutations of 3 relay-team members taken 3 at a time. To conserve time in writing this list on the chalkboard, the names may be abbreviated as follows.

L, B, H	B, L, H	H, L, B
L, H, B	B, H, L	H, B, L

KEY--PROBLEM SET 12:

1. 2
2. 1
3. 2
4. 2!
5. 2, 2
6. 5!
7. 120
8. 5,5
9. 3
10. 2
11. 1
12. 3! = 6
13. 9
14. $6 \cdot 5 = 30$
15. $10 \cdot 9 \cdot 8 = 720$
16. $n(n - 1)(n - 2) = n^3 - 3n^2 + 2n$
17. $(n - 2)(n - 3) = n^2 - 5n + 6$
18. $(n + 1)n = n^2 + n$
19. $10! = 720 \times 7!$
    $= 720 \times 5040$
    $= 3,628.800$
20. 4! = 24
21. 3! = 6
22. G,E,A E,G,A A,G,E
    G,A,E E,A,G A,E,G
23. $2 \cdot 1 \cdot 1 = 2$
    E,A,G A,E,G
24. 5! = 120
25. 6! = 720
26. 7! = 5040

LESSON 13: MORE PERMUTATIONS

OBJECTIVE:

The student will compute the number of permutations of n things taken r at a time.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

Section 13-1 contains a list of all permutations of 4 relay team members, taken 2 at a time. To conserve time in writing this list on the chalkboard, the names may be abbreviated as follows.

L, B	B, L	H, L	P, L
L, H	B, H	H, B	P, B
L, P	B, P	H, P	P, H

KEY--PROBLEM SET 13:

1. 3
2. 2
3. $3 \cdot 2 = 6$
4. 2
5. Linda, Bill
   Linda, Hortense
   Bill, Linda
   Bill, Hortense
   Hortense, Linda
   Hortense, Bill
6. $P(5,5) = 120$
7. $P(5,4) = 120$
8. $P(5,3) = 60$
9. $P(5,2) = 20$
10. $P(5,1) = 5$
11. $P(5,0) = 1$
12. 18,5
13. 360
14. 1
15. 9,900
16. 117,600
17. 7
18. 1
19. 3
20. $P(8,2) = 56$
21. $P(4,3) = 24$

22. $$P(17,2) \times P(8,1) = \frac{17!}{(17-2!)} \times \frac{8!}{(8-1)!} = \frac{17!}{15!} \times \frac{8!}{7!} = 17 \cdot 16 \cdot 8 = 2,176$$

23. $$P(17,2) \times P(8,2) = \frac{17!}{15!} \times \frac{8!}{6!} = 17 \times 16 \times 8 \times 7 = 15,232$$

24. $P(7,5) = 2520$

25. $P(5,3) = 60$

26. $P(5,2) = 20$

LESSON 14: COMBINATIONS

OBJECTIVE:

The student will compute the number of combinations of n objects taken r at a time.

PERIODS RECOMMENDED:

One

KEY--PROBLEM SET 14:

1. 5, 2
2. $\binom{5}{2}$
3. $\frac{5!}{2!3!}$
4. 10
5. L,B L,H
   L,P L,A
   B,H B,P
   B,A H,P
   H,A P,A
6. $\binom{11}{2} = 55$
7. $\binom{25}{3} = 2300$
8. $\binom{10}{3} = 120$
9. $\binom{10}{2} \cdot \binom{10}{2} = 45 \cdot 45$
   $= 2025$
10. a. 1
    b. 4
    c. 6
    d. 4
    e. 1
11. a. $\binom{4}{0}$
    b. $\binom{4}{1}$
12. a. 21
    b. 21
13. a. 1
    b. 4
    c. 2
    d. 200
14. $\binom{6}{3} = 20$
15. $\binom{10}{6} = 210$
16. $\binom{8}{3} = 56$
17. $\binom{52}{13} = \frac{52!}{13!39!}$
18. $\binom{4}{3}\binom{4}{2} = 4 \cdot 6$
    $= 24$

LESSON 15: COMBINATIONS IN GENETICS

OBJECTIVE:

The student will solve combination problems in genetics involving inherited traits.

PERIODS RECOMMENDED:

One

KEY--PROBLEM SET 15:

1. f f m
   f m f
   m f f
2. 3
3. 1, 3 or 3, 1
4. 2
5. $\binom{3}{2}$
6. 3
7. 0
8. $\binom{5}{0} = 1$
9.

number of females	0	1	2	3	4	5
number of sex patterns	1	5	10	10	5	1

10. $\binom{5}{1} = \frac{5!}{1!4!}$
    $\binom{5}{4} = \frac{5!}{4!1!}$

Therefore $\binom{5}{1} = \binom{5}{4}$

11.

number of fingers with hair	0	1	2	3	4
number of patterns	1	4	6	4	1

12. $\binom{8}{3} = 56$

13. $\binom{6}{4} = 15$

14. $\binom{6}{5} = 6$

15. $\binom{6}{6} = 1$

16. $\binom{4}{0} = 1$

17. $\binom{4}{1} = 4$

httt, thtt,
ttht, ttth

18. $\binom{4}{2} = 6$

hhtt, htht,
htth, thht,
thth, tthh

19. $\binom{4}{3} = 4$

hhht, hhth,
hthh, thhh

20. $\binom{4}{4} = 1$

LESSON 16: THE BINOMIAL THEOREM

OBJECTIVE:

The student will use the binomial theorem to solve combination problems.

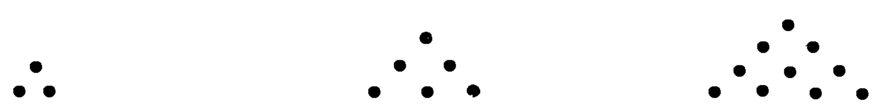
PERIODS RECOMMENDED:

Two

OVERVIEW AND REMARKS:

You may wish to extend the discussion of Pascal's Triangle by exploring some of the complex relationships that are embedded in it.

The first diagonal is made up of all 1's, the second diagonal is the set of natural numbers, and the third diagonal is the set of triangular numbers (numbers of the form $\frac{K(K + 1)}{2}$). Triangular numbers can be represented geometrically by triangular arrangements of dots. A few examples follow.



The triangular numbers are also the partial sums of the natural numbers. $3 = 1 + 2$, $6 = 1 + 2 + 3$, etc.

Note also that pairwise sums of the entries in the third diagonal give all the perfect squares.

$$1 + 3 = 4$$
$$3 + 6 = 9$$
$$6 + 10 = 16 \quad \text{etc.}$$

FIRST DIAGONAL
SECOND DIAGONAL
THIRD DIAGONAL

$$
\begin{array}{ccccccccccccccc}
 & & & & & & & 1 & & & & & & & \\
 & & & & & & 1 & & 1 & & & & & & \\
 & & & & & 1 & & 2 & & 1 & & & & & \\
 & & & & 1 & & 3 & & 3 & & 1 & & & & \\
 & & & 1 & & 4 & & 6 & & 4 & & 1 & & & \\
 & & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & & \\
 & 1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 & \\
1 & & 7 & & 21 & & 35 & & 35 & & 21 & & 7 & & 1
\end{array}
$$

Another fascinating pattern is that each number in the triangle is the sum of the entries in the diagonal to the left. The relationship is shown below.

$$
\begin{array}{ccccccccccccccc}
 & & & & & & & 1 & & & & & & & \\
 & & & & & & 1 & & 1 & & & & & & \\
 & & & & & 1 & & 2 & & 1 & & & & & \\
 & & & & 1 & & 3 & & 3 & & 1 & & & & \\
 & & & 1 & & 4 & & 6 & & 4 & & 1 & & & \\
 & & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & & \\
 & 1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 & \\
1 & & 7 & & 21 & & 35 & & 35 & & 21 & & 7 & & 1
\end{array}
$$

Notice that one case of the above observation is that the triangular numbers are the partial sums of the natural numbers, as indicated earlier.

KEY--PROBLEM SET 16:

1. $(f + m)^4 = (f + m)(f + m)^3$

$= (f + m)(f^3 + 3f^2m + 3fm^2 + m^3)$

$= f(f^3 + 3f^2m + 3fm^2 + m^3) + m(f^3 + 3f^2m + 3fm^2 + m^3)$

$= f^4 + 3f^3m + 3f^2m^2 + fm^3 + f^3m + 3f^2m^2 + 3fm^3 + m^4$

$= f^4 + 4f^3m + 6f^2m^2 + 4fm^3 + m^4$

2. a. 1 b. the same c. m d. f e. f

3. $(f + m)^6 = \binom{6}{6}f^6 + \binom{6}{5}f^5m + \binom{6}{4}f^4m^2 + \binom{6}{3}f^3m^3 + \binom{6}{2}f^2m^4 + \binom{6}{1}fm^5 + \binom{6}{0}m^6$

$= f^6 + 6f^5m + 15f^4m^2 + 20f^3m^3 + 15f^2m^4 + 6fm^5 + m^6$

4. n

5. $\binom{n}{n} = \frac{n!}{n!(n-n)!}$

$= \frac{n!}{n!0!}$

$= 1$

6. 0

7. $\binom{n}{0} = \frac{n!}{0!(n-0)!}$

$= \frac{n!}{0!n!}$

$= 1$

8. $27x^3 + 27x^2y + 9xy^2 + y^3$

9. $z^4 + 8z^3s + 24z^2s^2 + 32zs^3 + 16s^4$

10. $s^4 - 4s^3t + 6s^2t^2 - 4st^3 + t^4$

11. $27s^3 - 54s^2z + 36sz^2 - 8z^3$

12. $\frac{1}{27}x^3 + \frac{2}{9}x^2y + \frac{4}{9}xy^2 + \frac{8}{27}y^3$

13. $\frac{1}{27}x^3 - \frac{2}{9}x^2y + \frac{4}{9}xy^2 - \frac{8}{27}y^3$

14. $x^8 + 4x^6z^3 + 6x^4z^6 + 4x^2z^9 + z^{12}$

15.

$$\begin{array}{ccccccccccccccccc}
&&&&&&&&1&&&&&&&& \\
&&&&&&&1&&1&&&&&&& \\
&&&&&&1&&2&&1&&&&&& \\
&&&&&1&&3&&3&&1&&&&& \\
&&&&1&&4&&6&&4&&1&&&& \\
&&&1&&5&&10&&10&&5&&1&&& \\
&&1&&6&&15&&20&&15&&6&&1&& \\
&1&&7&&21&&35&&35&&21&&7&&1& \\
1&&8&&28&&56&&70&&56&&28&&8&&1
\end{array}$$

16. same as Problem 3.

17. a. 1 b. 2 c. 4 d. 8 e. 16

18. They are successive powers of 2.

19. 2^{60}

20. a. 2 b. 1 c. 3 d. 2 e. x

21. 15

22. a. $h^3 + 3h^2t + 3ht^2 + t^3$ b. 2, 1 c. 0, 3 d. 8

23. $\binom{n}{n-1}$

24. $\binom{n}{n-1} = \frac{n!}{(n-1)![n-(n-1)]!}$

$= \frac{n!}{(n-1)!1!}$

$= n$

25. $\binom{n}{1}$

26. $\binom{n}{1} = \frac{n!}{1!(n-1)!}$

$= n$

LESSON 17: REVIEW

OBJECTIVE:

The student will solve problems related to the objectives for Lessons 10 - 16.

PERIODS RECOMMENDED:

One

KEY--REVIEW PROBLEM SET 17:

1. $P(3,3) = 3! = 6$ 2. $P(6,3) = \frac{6!}{3!} = 6\cdot5\cdot4 = 120$ 3. $\binom{6}{3} = \frac{6!}{3!3!} = \frac{6\cdot5\cdot4}{3\cdot2\cdot1} = 20$

4. #2 is the permutation problem; #3 is the combination problem.

5. There are 3 ways to sentence Zeke and 2 ways to sentence each of his sons, so there are $3\cdot2\cdot2\cdot2\cdot2\cdot2 = 3\cdot32 = 96$ ways in all.

6. There are still 3 ways to sentence Zeke and 2 ways for each of his sons <u>except</u> Clem. There is only one way to sentence Clem. Thus there are $3\cdot2\cdot2\cdot2\cdot2\cdot1 = 3\cdot16 = 48$ ways in all.

7. a. $4^4 = 256$ b. $P(4,4) = 24$

8. a. $4^3 = 64$ b. $P(4,3) = 24$

9. a. $\binom{4}{2} = \frac{4!}{2!2!} = 6$ b. R,S R,T R,W S,T S,W T,W

10. $\binom{6}{2} = \frac{6!}{2!4!} = 15$

11. $(f + m)^6 = f^6 + 6f^5m + 15f^4m^2 + 20f^3m^3 + 15f^2m^4 + 6fm^5 + m^6$

12. $15f^2m^4$

13.

$$\begin{array}{ccccccccccccc}
 & & & & & & 1 & & & & & & \\
 & & & & & 1 & & 1 & & & & & \\
 & & & & 1 & & 2 & & 1 & & & & \\
 & & & 1 & & 3 & & 3 & & 1 & & & \\
 & & 1 & & 4 & & 6 & & 4 & & 1 & & \\
 & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & \\
1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1
\end{array}$$

14. a. 3 b. 6 c. 10 d. 20 e. 1 f. 1

15. $a^2 + 4ab + 4b^2$

16. $8z^3 - 12z^2s + 6zs^2 - s^3$

17. 840 19. 210 21. 604,800 23. 720

18. 35 20. 35 22. 120 24. 120

25. $\binom{8}{3} = 56$

26. $(m + g)^8 = m^8 + 8m^7g + 28m^6g^2 + 56m^5g^3 + 70m^4g^4 + 56m^3g^5 + 28m^2g^6 + 8mg^7 + g^8$

27. $56m^3g^5$ 28. $8\cdot7\cdot6 = 336$ 29. $3\cdot2\cdot3 = 18$

LESSON 18: PROBABILITY

OBJECTIVES:

The student will:

compute the probabilities of events from relative frequencies.

compute the combined probability of two or more independent events.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

In this lesson and in Lesson 19 slide rules or calculators can be used to special advantage. There are numerous problems involving probability multiplications.

The problem set contains problems on mating patterns with respect to race and religion. This subject could be extended into the Social Science class, if desired.

KEY--PROBLEM SET 18:

1\. a. 52

b. 13

c. 4

d. 20

2\. $\frac{13}{52} = \frac{1}{4}$

3\. $\frac{4}{52} = \frac{1}{13}$

4\. $\frac{20}{52} = \frac{5}{13}$

5\. $\frac{1}{13}$

6\. $\frac{8}{13}$

7\. $\frac{1}{13}$

8\. $\frac{1}{13}$

9\. $\frac{1}{13} \times \frac{1}{13}$

10\. $\frac{1}{169}$

11\. $\frac{2}{169}$

12\. $\frac{3}{13} \times \frac{3}{13} = \frac{9}{169}$

13\. $\frac{8}{13} \times \frac{8}{13} \times \frac{8}{13} \times \frac{8}{13} = \frac{4096}{28561}$

14\. $\frac{6}{25} = .24$

15\. .24

16\. .12

17\. .25

$$\left.\begin{array}{ll} 18. & .12 \times .25 = .03 \\ 19. & .88 \times .66 \approx .581 \\ 20. & .12 \times .66 \approx .079 \end{array}\right\} \quad \text{See tables below for actual census values.}$$

The following tables facilitate a comparison between predicted and actual relative frequencies of the sort calculated in Problems 18 through 20.

PREDICTED ON INDEPENDENCE ASSUMPTION

	White	Non-white
Protestant	.581	.079
Roman Catholic	.220	.030
Other or none	.079	.011

ACTUAL RELATIVE FREQUENCIES

	White	Non-white
Protestant	.569	.096
Roman Catholic	.248	.007
Other or none	.073	.009

The predicted and actual figures are not in agreement. A statistical test, such as chi square, would verify that the differences are significant. Therefore the independence assumption must be rejected. The higher than predicted relative frequency of non-white Protestants is a reflection of the southern origins of the black population. The South is more strongly Protestant than the country as a whole. Another factor is extensive immigration of white Catholics from Europe.

21. $.11 \times .88 \approx .097$ This answer can be explained as follows. Suppose mating is random with respect to race and consider a randomly chosen married couple. Looking at the male partner, the probability that he will be black is .11. This is true because the relative frequency of blacks among males is assumed to be the same as that of blacks in the entire population. Similarly the probability that the female partner will be white is .88. This leads to the probability product above.

22. $.88 \times .11 \approx .097$ The actual figures are naturally much lower. The total rate of black-white intermarriage has run as high as 5% in some localities. This falls far short of the 9.7% predicted above for just one type of black-white intermarriage. More on this in the next lesson.

23. $.66 \times .25 = .165$ (The actual figure is about .055.)

24. .0015   25. .023   26. .2116   27. .0138

LESSON 19: MUTUALLY EXCLUSIVE EVENTS

OBJECTIVE:

The student will compute the combined probability of two or more mutually exclusive events by summing the individual probabilities.

PERIODS RECOMMENDED:

One

KEY--PROBLEM SET 19:

1. .30
2. $.03 + .67 = .70$
3. $.30 + .67 = .97$
4. .04
5. $.20 + .36 + .04 = .60$
6. $1 - .28 = .72$
7. $.36 + .04 = .40$
8. Roman Catholic man, Protestant woman
9. yes
10. sum
11. $.66 \times .25 = .165$
12. $.25 \times .66 = .165$
13. $.165 + .165 = .33$

14.

$$\begin{array}{ll} P(\text{white, black}) = 2(.88)(.11) & \approx .194 \\ P(\text{white, other}) = 2(.88)(.01) & \approx .018 \\ P(\text{black, other}) = 2(.11)(.01) & \approx .002 \\ \hline P(\text{intermarriage}) & \approx .214 \end{array}$$

So around 21% intermarriage would be expected on the basis of random mating.

LESSON 20: PROBABILITIES OF TRAIT PATTERNS

OBJECTIVE:

The student will compute the probabilities of various sex patterns and other biological traits.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

In the text, brackets are used to help avoid confusing powers of fractions and binomial coefficients. For example, we write $\binom{7}{3}\left[\frac{3}{4}\right]^7$ rather than $\binom{7}{3}\left(\frac{3}{4}\right)^7$.

KEY--PROBLEM SET 20:

1. ff,fm,mf,mm
2. $\frac{1}{2}, \frac{1}{2}$
3. $\frac{1}{2}, \frac{1}{2}$
4. $\frac{1}{4}$
5. $\frac{1}{4}$
6. fm mf
7. $\frac{1}{4}$
8. $\frac{1}{4}, \frac{1}{4}$
9. $\frac{1}{2}$
10. $\binom{4}{1}$ or $\binom{4}{3}$
11. 4
12. fffm,ffmf,fmff,mfff
13. $\left[\frac{1}{2}\right]^4 = \frac{1}{16}$
14. $4 \times \frac{1}{16} = \frac{1}{4}$
15. a. 15

    b. $\frac{1}{64}$

    c. $15 \times \frac{1}{64} = \frac{15}{64}$
16. $\binom{7}{5} \times \left[\frac{1}{2}\right]^7 = \frac{21}{128}$
17. a. $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$

    b. $\frac{1}{64}$
18. a. $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$

    b. $\frac{27}{64}$
19. a. $\binom{3}{2} = 3$

    b. $\left[\frac{1}{4}\right]^2\left[\frac{3}{4}\right] = \frac{3}{64}$

    c. $\frac{9}{64}$
20. $\frac{135}{512}$

LESSON 21: THE BINOMIAL THEOREM REVISITED

OBJECTIVE:

The student will use the Binomial Theorem to compute the probability that certain patterns of traits will occur.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

The approach used in this lesson is that of writing an entire binomial expansion and then selecting the terms appropriate for a given problem. If you wish to spend more time, you could also discuss the procedure for writing only the relevant terms. (See Problem 17 in the problem set).

KEY--PROBLEM SET 21:

1. $(f + m)^2 = f^2 + 2fm + m^2$

2. $2\left[\frac{1}{2}\right]\left[\frac{1}{2}\right]$

3. $\frac{1}{2}$

4. $(f + m)^4 = f^4 + 4f^3m + 6f^2m^2 + 4fm^3 + m^4$

5. 1

6. $4fm^3$

7. $4\left[\frac{1}{2}\right]\left[\frac{1}{2}\right]^3$

8. $\frac{1}{4}$

9. $\frac{1}{4} + \frac{1}{16} = \frac{5}{16}$

10. $(a + n)^3 = a^3 + 3a^2n + 3an^2 + n^3$

$$3a^2n = 3\left[\frac{1}{4}\right]^2\left[\frac{3}{4}\right] = \frac{9}{64}$$

11. $\left[\frac{1}{4}\right]^3 + \left[\frac{3}{4}\right]^3 = \frac{1}{64} + \frac{27}{64} = \frac{28}{64} = \frac{7}{16}$

12. $\left[\frac{1}{4}\right]^4 + \left[\frac{3}{4}\right]^4 = \frac{41}{128}$

13. $4\left[\frac{1}{4}\right]\left[\frac{3}{4}\right]^3 + \left[\frac{3}{4}\right]^4 = \frac{189}{256}$

14. This situation includes exactly those cases not included in Problem 13. Therefore

$$p = 1 - \frac{189}{256} = \frac{67}{256}$$

15. Let B stand for brown and b stand for blue.

$$(B + b)^5 = B^5 + 5B^4b + 10B^3b^2 + 10B^2b^3 + 5Bb^4 + b^5$$

$$10B^2b^3 = 10\left[\frac{3}{4}\right]^2\left[\frac{1}{4}\right]^3 = \frac{45}{512}$$

16. $B^5 + 5B^4b = \left[\frac{3}{4}\right]^5 + 5\left[\frac{3}{4}\right]^4\left[\frac{1}{4}\right]$

$$= \left[\frac{3}{4}\right]^4\left[\frac{3}{4} + \frac{5}{4}\right] = 2\left[\frac{3}{4}\right]^4 = \frac{81}{128}$$

17. $\binom{20}{15}\left[\frac{1}{2}\right]^{20}$ or $\binom{20}{5}\left[\frac{1}{2}\right]^{20}$

If any students are interested, this expression equals

$$\frac{969}{65,536} \approx .015$$

LESSON 22: GENES AND INHERITANCE

OBJECTIVE:

The student will compute the probabilities of various kinds of offspring, given the genetic types of the parents.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

See the Teacher Introduction in Lesson Plan 1 for remarks on pacing with Science. If Science has not completed Lesson 16 by this time, you will need to introduce and explain some vocabulary before starting on this lesson. You will find the necessary material in Science Lesson 16. The essential terms are as follows.

gene	homozygous
genotype	heterozygous
phenotype	dominant gene
recessive gene	

KEY--PROBLEM SET 22:

1. clockwise
2. clockwise
3. counterclockwise
4. D, d
5. d
6. $\frac{1}{2}$
7. $\frac{1}{2}$
8. 0
9. 1
10. $\frac{1}{2} \times 1$
11. $\frac{1}{2}$
12. $\frac{1}{2} \times 1$
13. $\frac{1}{2}$
14. $\frac{1}{2}Dd + \frac{1}{2}dd$
15. $\frac{1}{2}$
16. $\frac{1}{2}$
17. A
18. AB
19. I^AI^A, I^AI^B, I^OI^A, I^OI^B
20. $\frac{1}{4}$
21. A, B, AB
22. $\frac{1}{2}$
23. I^BI^O
24. The child does not carry an I^B gene. If it did it could not have blood type A.
25. I^BI^O
26. I^AI^O
27. I^AI^B, I^AI^A, I^AI^O
28. AB, A
29. I^AI^O
30. I^BI^O
31. B
32. AB, A
33. The movie star could not have been the father. He does not carry an I^B gene, nor does the mother. Since the child carries an I^B gene, the father must have been someone else.
34. I^AI^B
35. a. I^BI^O b. I^AI^O, I^AI^A c. A

LESSON 23: GENE FREQUENCIES

OBJECTIVE:

The student will compute gene frequencies for various genetic traits.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

The M-N blood types are introduced here and used in various problems. Be sure to emphasize that this blood typing system is distinct from the A-B-O system.

KEY--PROBLEM SET 23:

1. # of S genes, total # of genes
2. The chimp can't taste PTC. The others can.
3. 6
4. 2
5. 4
6. $\frac{1}{3}$
7. $\frac{2}{3}$
8. a. 361

   b. 722
9. $(2 \cdot 305) + 52 = 662$
10. $\frac{662}{722} = \frac{331}{361}$

    $\simeq .92$
11. $(2 \cdot 4) + 52 = 60$
12. $\frac{60}{722} = \frac{30}{361}$

    $\simeq .08$
13. $I^M I^M$
14. .50
15. $f(I^M) \simeq .23 + \frac{1}{2}(.50)$

    $\simeq .48$
16. $f(I^N) \simeq .27 + \frac{1}{2}(.50)$

    $\simeq .52$
17. $f(T) \simeq .17 + \frac{1}{2}(.48)$

    $\simeq .41$
18. $f(T) \simeq .20 + \frac{1}{2}(.50)$

    $\simeq .45$
19. Eskimo
20. $f(I^A) \simeq .01 + \frac{1}{2}(.18)$

    $\simeq .10$
21. $f(I^B) \simeq 0$
22. $f(I^O) \simeq .81 + \frac{1}{2}(.18)$

    $\simeq .90$
23. I^B

LESSON 24: GENETIC PREDICTIONS

OBJECTIVE:

Given the gene frequencies of the present generation, the student will compute the genotype frequencies in the next generation.

PERIODS RECOMMENDED:

One

--PROBLEM SET 24:

.40

.40

a. .40, .40

b. .16

a. .60, 60

b. .36

.16
.24
.24
.36

6. tasters

7. .64TT + .32Tt + .04tt

8. a. .64

b. 64

9. 32

10. 4

11. 96

12. $.90I^M + .10I^N$

13. $.81I^MI^M + .18I^MI^N + .01I^NI^N$

14. M--.81
MN--.18
N--.01

15. Eskimos ≈ 16% [2(.09)(.91)]
Aborigines ≈ 30% [2(.18)(.82)]

Incidence will be higher among Aborigines.

;ON 25: THE HARDY-WEINBERG LAW

:CTIVE:

The student will combine the concepts of the two preceding lessons to compute relative frequencies of genotypes in one generation, given the relative frequen- ; of genotypes in the preceding generation.

:ODS RECOMMENDED:

One

{VIEW AND REMARKS:

The Hardy-Weinberg Law states that under certain conditions, including random .ng, the relative frequencies of genotypes for a trait remain constant from one :ration to the next. The mathematical development in this lesson has another .ication, which is not stated explicitly. Regardless of the relative frequen- ; of the genotypes, if mating is random, an equilibrium will be attained after generation. Such a situation could occur in the laboratory if a population : gathered from several gene pools and then combined and isolated.

-PROBLEM SET 25:

80%

$.40 + \frac{1}{2}(.40) = .60$

$.20 + \frac{1}{2}(.40) = .40$

.60D + .40d

.36DD + .48Dd + .16dd

.36 .48 .16

decreased

8. $f(D) = .36 + \frac{1}{2}(.48)$
$= .60$
$f(d) = .16 + \frac{1}{2}(.48)$
$= .40$

9. DD - .36
Dd - .48
dd - .16

10. same

11. They will be the same.

12. TT - .09
Tt - .42
tt - .49

13. same

14. 60.84%

LESSON 26: REVIEW

OBJECTIVE:

The student will solve problems related to the objectives for Lessons 18 - 25.

PERIODS RECOMMENDED:

One

KEY--REVIEW PROBLEM SET 26:

1. a. $\frac{1}{6}$
   b. $\frac{1}{2}$
   c. $\frac{1}{2}$
2. a. $\frac{1}{13}$
   b. $\frac{1}{4}$
   c. $\frac{1}{52}$
3. .20
4. .40
5. .50
6. .70
7. .24
8. .08
9. .54
10. .24
11. .08
12. .48
13. .24
14. $\frac{9}{256}$
15. $\frac{1}{256}$
16. $n^4 + 4n^3s + 6n^2s^2 + 4ns^3 + s^4$
17. $\frac{27}{128}$
18. $\frac{81}{256}$
19. $\frac{243}{256}$
20. I^AI^O, I^BI^O
21. A, B
22. I^BI^O
23. I^AI^O
24. B, AB
25. .58
26. .42
27. .70
28. .30

29.

	GENOTYPE	RELATIVE FREQUENCY
GENERATION II	BB	.49
	Bb	.42
	bb	.09

30. Same as #29

COMMON LOGARITHMS

y	log y	y	log y
1.0	.000	5.5	.740
1.1	.041	5.6	.748
1.2	.079	5.7	.756
1.3	.114	5.8	.763
1.4	.146	5.9	.771
1.5	.176	6.0	.778
1.6	.204	6.1	.785
1.7	.230	6.2	.792
1.8	.255	6.3	.799
1.9	.279	6.4	.806
2.0	.301	6.5	.813
2.1	.322	6.6	.820
2.2	.342	6.7	.826
2.3	.362	6.8	.833
2.4	.380	6.9	.839
2.5	.398	7.0	.845
2.6	.415	7.1	.851
2.7	.431	7.2	.857
2.8	.447	7.3	.863
2.9	.462	7.4	.869
3.0	.477	7.5	.875
3.1	.491	7.6	.881
3.2	.505	7.7	.886
3.3	.519	7.8	.892
3.4	.531	7.9	.898
3.5	.544	8.0	.903
3.6	.556	8.1	.908
3.7	.568	8.2	.914
3.8	.580	8.3	.919
3.9	.591	8.4	.924
4.0	.602	8.5	.929
4.1	.613	8.6	.934
4.2	.623	8.7	.940
4.3	.633	8.8	.944
4.4	.643	8.9	.949
4.5	.653	9.0	.954
4.6	.663	9.1	.959
4.7	.672	9.2	.964
4.8	.681	9.3	.968
4.9	.690	9.4	.973
5.0	.699	9.5	.978
5.1	.708	9.6	.982
5.2	.716	9.7	.987
5.3	.724	9.8	.991
5.4	.732	9.9	.996